1. (a) Evaluate $\langle n | x^n | n \rangle$. Show that $\langle m | x^3 | n \rangle = 0$ unless $|m-n|=1$ or 3.

2. Calculate $Z$ for the harmonic oscillator. Calculate $\bar{n} = \sum_n n P_n$, the mean level occupancy, where $P_n$ is the probability that the $n^{th}$ level is occupied. Find the first two terms of the high T expansion of $\bar{n} + 1/2$.

3. For CO$_2$, only the z-polarized vibrations were discussed in lecture. What are the x-polarized eigenvectors, and what can you say about their eigenvalues?

4. For the 6-atom monatomic linear chain, (a) what is the displacement pattern at $k=\pi/a$? At $k=-\pi/a$? (b) The smallest non-zero k-vectors (for 6 atoms) are $k=\pi/3a$ and $k=-\pi/3a$. The corresponding eigenvectors of D are complex. What about the real linear combinations, $(\pi/3a) + [-\pi/3a]/\sqrt{2}$ and $(\pi/3a) - [-\pi/3a]/\sqrt{2}i$? Do they give legal solutions of Newton’s laws? What are the displacement patterns?

5. The diatomic linear chain was described briefly in lecture. Derive the 2 x 2 dynamical matrix $D(Q)$ in $Q$ (or $k$)-space. Find the eigenvalues. What happens when $M_1=M_2$? What is the sound velocity and how does it compare with the monatomic chain?