Pure appl. geophys. 166 (2009) 1885–1908
0033–4553/09/101885–24
DOI 10.1007/s00024-009-0520-3

Pure and Applied Geophysics

Landslides, Ice Quakes, Earthquakes: A Thermodynamic Approach to Surface Instabilities

KLAUS REGENAUER-LIEB,[1,2] DAVID A. YUEN,[3] and FLORIAN FUSSEIS[1]

Abstract—The total rate of rock deformation results from competing deformation processes, including ductile and brittle mechanisms. Particular deformation styles arise from the dominance of certain mechanisms over others at different ambient conditions. Surprisingly, rates of deformation in naturally deformed rocks are found to cluster around two extremes, representing coseismic slip rates or viscous creep rates. Classical rock mechanics is traditionally used to interpret these instabilities. These approaches consider the principle of conservation of energy. We propose to go one step further and introduce a nonlinear far-from-equilibrium thermodynamic approach in which the central and explicit role of entropy controls instabilities. We also show how this quantity might be calculated for complex crustal systems. This approach provides strain-rate partitioning for natural deformation processes occurring at rates in the order of 10^{-3} to 10^{-9} s^{-1}. We discuss these processes using examples of landslides and ice quakes or glacial surges. We will then illustrate how the mechanical mechanisms derived from these near-surface processes can be applied to deformation near the base of the seismogenic crust, especially to the phenomenon of slow earthquakes.

Key words: Seismology, geodynamics, instabilities, thermodynamics, entropy production, numerical modelling.

Notation

Variable

S	*Entropy*
s	*Specific entropy*
Q	*Heat*
T	*Absolute temperature*
ρ	*Density*
$e_{tot,\ kin}$	*Specific total, kinetic energy*
u	*Specific internal energy*
V	*Volume of the reference volume element (RVE)*
A	*Surface of the RVE*
σ_{ij}	*Total Cauchy stress tensor*
ε_{ij}	*Total strain tensor*
q_i	*Surface heat flow, radiative and conductive*

[1] Multi-scale Earth System Dynamics, School of Earth and Environment, University of Western Australia, Crawley, WA 6009, Australia. E-mail: klaus.regenauer-lieb@csiro.au; fusseis@cyllene.uwa.edu.au
[2] CSIRO Exploration and Mining, PO Box 1130, Bentley, WA 6102, Australia.
[3] Department of Geology and Geophysics and Minnesota Supercomputing Institute, University of Minnesota, Minneapolis, MN 55455, U.S.A.

ψ	*Specific Helmholtz free energy*
ε_{ij}^{el}	*First state variable, the total elastic strain tensor*
α_k	*Other state variables (e.g. total creep strain tensor)*
C_α	*Specific heat for constant* α
λ	*Thermal expansion coefficient*
p	Pressure
ξ	*Fractional volume of one phase in a two phase mixture*
χ	*Taylor-Quinney heat conversion efficiency*
L	*Latent heat release*
κ	*Thermal diffusivity*

1. Introduction: Classical Rock Mechanics and Thermodynamics Approaches

Here we present the hypothesis that far-from-equilibrium, nonlinear thermodynamics provides a unified framework for earthquake modeling. The rapid movements of large masses of the crust have a direct impact on human communities and a deep understanding of these phenomena requires researchers to take a fresher view into the dynamical properties of rock deformation near the surface. The strain energy of a deforming rock is generally released through a number of different creep mechanisms. These deformation mechanisms span a wide range and involve elastic straining, dislocation- and diffusion creep, metamorphic reactions, phase transitions, chemical reactions, fluid-rock interaction, cleavage fracturing, cavitation and frictional sliding.

Observational evidence for the different mechanisms come from a wide range of disciplines, from extrapolation of experimental data, microstructure, microgeochemistry (isotope measurements, 'geospeedo-metry'), remote sensing (GPS/InSAR) and geophysical measurements (borehole strainmeters). Total rates of natural deformation processes have been bracketed (Fig. 1) either at seismic rates (e.g., fracture propagation, frictional sliding/melting, which happens within seconds) or at strain rates of 10^{-10} to 10^{-15} s^{-1} where point/line-defect diffusion controls the viscous (i.e., temperature-activated) response of a rock. A geological system where deformation takes place at all strain rates is the seismogenic crust. Deformation occurs on the time scale of the earthquake cycle on the order from tens to hundreds of years. During interseismic periods various 'slow' diffusion processes ultimately control most deformation, whereas during earthquakes 'rapid', unstable brittle processes control the deformational response of the fault to coseismic stress release.

Consequently, during the earthquake cycle, deformation rates vary over about 14 orders of magnitude. Within the earthquake cycle "slow earthquakes" release strain energy over a time scale of several hours to days, often as a precursor to larger earthquakes (e.g., LINDE *et al.*, 1996). This time scale suggests that slow earthquakes happen at intermediate rates between those achieved during interseismic and coseismic periods.

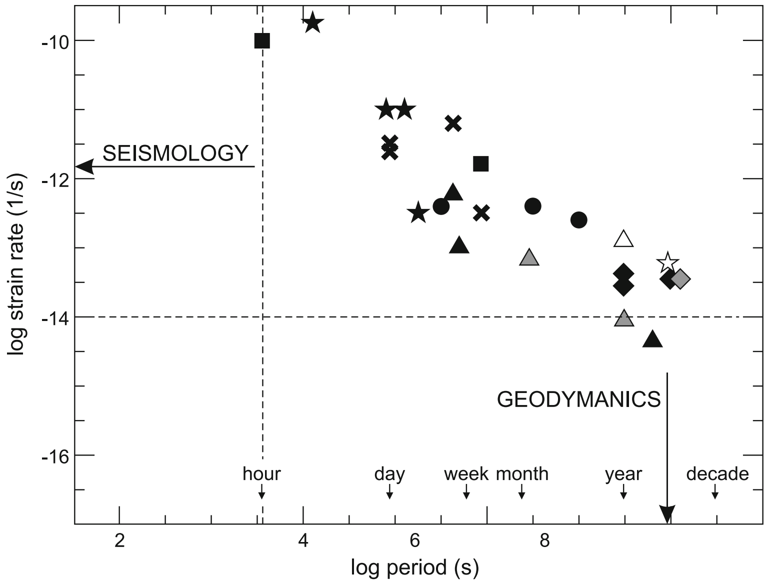

Figure 1
Measured strain rates by: GPS (gray), InSAR(white), Strainmeters (black). Triangles show postseismic deformation from the Landers earthquake, squares show slow earthquakes, open squares single station observations. GPS and InSAR data have higher sensitivity at long periods (months to decades) while borehole strain meters are better at short periods. While some data are derived close to their detection limit, the plots illustrate the point that there is no separation between the classical geodynamic strain rates (dashed line plate motion) and seismological strain rate. Geological deformation takes place at all rates. Source: PBO Steering Committee, The Plate Boundary Observatory: Creating a four-dimensional image of the deformation of western North America, White paper providing the scientific rationale and deployment strategy for a Plate Boundary Observatory based on a workshop held October 3-5, 1999, modified after: http://www.nap.edu/books/0309065623/xhtml/images/p20007dc2g215001.jpg.

In classical rock mechanics widely different micromechanical deformation mechanisms are used to explain the wide range of strain-rates observed in nature. However, the processes are normally not treated in a fully coupled way. There also does not exist a unified theory linking the scales. Rock mechanical properties are assigned to a given micromechanical group (see Appendix A). Interactions are often parameterized in experimentally derived state-dependent laws considering averaging variables (RICE 1971). In earthquake modeling, the use of experimentally constrained (DIETERICH, 1979a, b) rate and state-dependent friction law has proven to be very useful (TSE and RICE, 1986). These rock-mechanical descriptions are in themselves very similar to an isothermal thermomechanic approach but they are normally not derived from thermomechanics *a priori*, see ZIEGLER (1983) for a full thermomechanic approach. This work has now progressed further and an excellent approach to plasticity theory based on thermodynamic principles has been published recently (HOULSBY and PUZRIN, 2007). We argue, however, to go one step further for earthquake modeling. Such full nonisothermal thermodynamic formulations have only been available since the early 90's, being originally motivated by the advent of thermal infrared imaging of deformation processes (Chrysochoos and Dupre, 1991). The approach has, however, a huge potential beyond this application. It offers an extension of thermomechanics into the nonisothermal domain. The approach also offers the possibility to formulate a unified multi-scale thermodynamical framework for coupling mechanical and chemical simulations (Poulet and Regenauer-Lieb, 2009; Rambert *et al.*, 2007).

For earthquake modeling the clear advantage is that dynamic properties of rocks are not tied to specific experiments but property variations can be predicted in a truly time- or state- dependent manner based on the rate of entropy production. In this contribution we will elaborate further what this theory might hold for the application to earthquake modeling. We relate this concept to earlier case studies of thermal-mechanical instabilities, flowing of large ice sheets and unstable landslides.

2. *Instabilities and their Mathematical Expression*

Clausius (1865) introduced the concept of entropy as a simplifying concept for the understanding of thermal processes and not as a complication. Quite the opposite appears to be the case today. In geosciences we seem to have forgotten Clausius's message and try to avoid entropy production as a simplifying method in our calculations.

There is considerable confusion about entropy in the present literature. Entropy appears to be a concept that is only understood by information theorists or theoretical physicists/chemists. In the following we try to describe it as a simplifying concept that allows us to understand geomechanical problems with a thermodynamic foundation. In this attempt we avoid to go into subjects such as stochastic geometry and information theory (Attard, 2006) and focus on the potential application to earthquake simulation. In its most basic form the definition of entropy was put forward in order to formalize the simple concept that heat flows spontaneously from a hot to a cold object. No process can take place whose net effect is only to transfer heat from a cold object to a hot object. We hence revisit Clausius original definition by which entropy is defined as a quantity to describe what was later known as the second law of thermodynamics.

$$dS \geq \frac{\delta Q}{T}, \tag{1}$$

where S is the entropy of the system, d represents an infinitesimally small change of a state function, δ represents that of a path function. Q is the heat and T the absolute temperature. Maximum entropy defines a thermodynamic system at equilibrium. This entropy state is used later for the definition of a representative volume element (RVE). For reversibility the inequality in turns into equality. We also introduce the specific entropy which is

$$dS = \int_V \rho s \, dV. \tag{2}$$

Now consider a representative volume element in the current configuration and thermodynamic equilibrium. Its internal energy in motion is the sum of its specific internal energy u plus its specific kinetic energy e_{kin}.

$$\int_V \rho \, e_{tot} \; dV = \int_V \rho \, u \; dV + \int_V \rho \; e_{kin} \, dV \tag{3}$$

In the following we use the classical approach of creeping flows conveniently used for geodynamic modeling. We neglect the effect of the kinetic energy. This clearly does not apply to earthquake mechanics, however, as a first step towards modeling the conditions leading to an earthquake we wish to investigate competing strain rate processes prior to the development of kinetic energy. As a caveat we emphasize that this approach is meaningful only for investigation of the early mechanisms underlying the physics of slow earthquakes, ice quakes and landslides.

2.1. Local Equilibrium: The Small Scale

The time scale of interest defines the choice of the suitable size of the representative local equilibrium volume element v in a material reference frame (current volume) for a given observation time. This is the crucial step in the thermodynamics equilibrium assumption. Consider two thermodynamics processes happening at two very different time and length scales, i.e., the time t_1 to reach local equilibrium in the reference volume considerably smaller than the size of the system under study and the time t_2 required to reach the equilibrium in the entire system, $t_1 << t_2$. Following Onsager's regression hypothesis (ONSAGER, 1931) the time evolution of the fluctuation of a given physical value in an equilibrium system obeys the same laws on the average as the change of the corresponding macroscopic variable in a nonequilibrium system.

For the particular time scale of interest we define the smallest volume that allows a calculation of an average continuum property (RICE 1971) as the arbitrary reference volume in local thermodynamic equilibrium. Even at this smallest scale of the RVE we place ourselves in the framework of continuum mechanics where every point of the material can itself be imaged as a continuum. In other words, we assume that the RVE contains a sufficient number of discrete entities such that the laws of thermodynamics apply. This approach can be applied to all scales down to microscale. However, below nanoscale, statistical mechanics would replace the approach presented here. Emergent equilibrium properties of the continuum systems are embedded in a larger system that is not in equilibrium. We obtain for negligible kinetic energy the following relation for the RVE:

$$\int \dot{e}_{tot}\, dV = \int \dot{u}\, dV, \tag{4}$$

where, for convenience, the overdot denotes differentiation with respect to the Lagrangian, material time derivative. For this reference volume the first law of thermodynamics spells out the energy conservation under the assumption of small perturbations, conservation of mass and creeping flow

$$\int_V \rho\dot{u}\, dV = \int_V \sigma_{ij}\dot{\varepsilon}_{ij}\, dV + \int_V r_i\, dV - \int_A q_i\, dA. \tag{5}$$

We assume Einstein's summation convention and identify σ_{ij} as the total Cauchy stress tensor, which is work conjugate to the symmetric strain tensor ε_{ij}. In continuum mechanics work conjugacy implies that the product of the stress with the strain increments gives the total rate of work input to the material, per unit volume. We are using symmetric tensors because these follow from moment equilibrium of the continuum. In a first attempt to use thermodynamics for linking earthquake modeling with geodynamics we do not use the theory of micropolar continua because nonsymmetric tensor formulations would be required. Conservation of energy is achieved if the rate of external work expended on the reference volume is equal to the mechanical work, including the heat produced in the reference volume (first term on the right-hand side) plus a term related through internal heat r_i generation such as radioactive decay, Joule heating or heat generation through chemical reactions (second term on the right-hand side) minus the radiative and conductive heat transfer q_i on the surface of the volume respectively (third term).

Note that the additional heat term r_i appears as a source term on the right-hand side if e.g., radioactive decay is considered without considering its effect on the specific internal energy u. Such a loose formalism may appear convenient (POULET and REGENAUER-LIEB, 2009) if the scale of the radioactive isotopes is not resolved in the mathematical treatment. However, we emphasize that this loose formalism is strictly seen as not satisfactory in a more general approach (HOULSBY and PUZRIN, 2007).

The fundamental thermodynamic energy balance for the specific energy is given in terms of the specific entropy s by

$$u = \psi(T, \varepsilon_{ij}^{el}\{\alpha_k\}) + sT, \tag{6}$$

where ψ is the specific Helmholtz free energy and the state variables are the elastic strain ε^{el} and the absolute Temperature T and α_k other state variables including for instance the plastic strains caused by the individual micromechanical deformation mechanisms.

This equation describes the system in thermodynamic equilibrium given the following condition. *Equilibrium is achieved if the entropy goes to a maximum.* As a universal principle we hence use a strong form of the second law of thermodynamics. i.e. rather than just implying the second law of thermodynamics as a criterion for the direction of heat flow (from hot to cold, therefore the entropy must be positive) we require it to reach a maximum. Maximum entropy implies that the conjugate quantity to entropy, i.e., the stored energy (Helmholtz free energy, Gibbs free energy) goes to a minimum at equilibrium. Therefore for the small volume element we can derive equilibrium material properties such as elastic properties simply from a chemical Gibbs free energy minimizer (SIRET *et al.*, 2008). Note that time-dependent processes do not explicitly enter the discussion yet as we are still discussing equilibrium thermodynamics and time is irrelevant.

In order to illustrate this concept consider the following example: The RVE is a purely isothermal conductive solid with initial thermal insulation at all boundaries. At time $t_0 << t_1 << t_2$ the top and bottom thermal boundaries are removed and the RVE is subject to higher constant temperature from below and a lower constant temperature on the top boundary. Heat flows through the RVE and it is not in equilibrium. According to our choice this RVE can be considered in equilibrium only when it has reached its maximum entropy (linear thermal gradient) at time t_1. There is no more time dependency in the system. We can derive material parameters for this RVE for time scale t_2 by solving for the minimum Helmholtz free energy. Having defined the thermodynamic equilibrium time/length scale we may now wish to proceed to the nonequilibrium processes at the large scale. In the following we will implicitly assume integration over the RVE and drop the volume integral.

2.2. *Nonequilibrium Assumption: The Large Scale*

For the large nonequilibrium system we need to know how the macroscopic system responds over some reference time to small fluctuations of a given physical value at the local equilibrium. The small reference volume is at equilibrium. Equation (6) is appropriate for the small volume element but insufficient for describing the large nonequilibrium system. For this we have to consider the time evolution for which we differentiate Equation (6) with respect to time

$$\dot{u} = \dot{\psi}(T, \varepsilon_{ij}^{el}, \alpha_k) + \dot{s}T + \dot{T}s. \tag{7}$$

The second term now describes the rate of entropy production and the third term the coupled variation of temperature with time. Feedback between the three terms on the right-hand side of Equation (7) underpins the localization phenomena discussed in this paper. This feedback expresses itself as a competition of rates of processes, which happen at vastly different time and length scales. Equation (7) describes a rigorous and consistent framework, within which models can be developed, to describe a wide range of material processes, which we believe to be important for explaining the strain rate cascade in Figure 1.

Equation (7) relates the rate of internal energy to the sum of the time-rate of the Helmholtz free energy plus the rate of the entropy production composed of mechanical and thermal dissipation processes. Feedback between any of the terms can lead to instability. As an example, feedback between thermal and mechanical dissipation processes leads to the well-known thermal runaway instability (e.g., OGAWA, 1987) for instance if production of heat is faster than loss of heat due to diffusion. However, the thermal runaway process is only one of many possible feedbacks. In more general terms Equation (7) defines for the general case, conditions for instability if some critical internal parameter is exceeded. Shear heating may be the first and most obvious feedback mechanism in the strain-rate cascade of Figure 1 but we show later on in the Landslide example that it is overtaken by another critical mechanism once the strain rates have increased sufficiently above geodynamic rates. In the following we highlight the important difference of this nonlinear far-from-equilibrium thermodynamic approach formalized in Equation (7) with theories for localization in the classical rock mechanics formulation (Appendix A) and Prigogines localization theories for chemistry (next section).

Classical rock mechanics deals with isothermal deformation and Equation (7) simplifies to the theory of thermomechanics (see Appendix A). There is no thermal dissipation and this important feedback is suppressed. The time dependence of thermal diffusion is also missing and, if there is no kinetic energy and no chemical diffusion, then thermomechanics reverts to the classical theory of plasticity with a check on thermodynamic consistency. In this case the internal energy rate (rate of mechanical work applied to volume element) is equal to the rate of change of Helmholtz free energy (stored energy) plus the dissipation (entropic work rate or dissipated part of the plastic work rate). Because the time dependence is lost, the entropy must be formulated along a new quasi-steady state upon reaching the plastic yield stress. ZIEGLER (1983) shows that this leads to the principle of *maximimum entropy production* as a thermomechanics consistency check for continuum mechanics. Maximum entropy production is a strong assumption and it is only achieved if the nonlinear thermal mechanical process has reached some form of steady state, i.e., temperature does not change. This assumption is hence not very useful for earthquake modeling and we recommend use of the entire formulation in Equation (7).

2.3. Linear Stability and Localization

A localization phenomenon can be understood mathematically as a bifurcation phenomenon where the velocity field of a smoothly deforming/reacting solid aborts the continuous branch and takes a new discontinuous path. Equation (7) can be analyzed using classical concepts of linear stability and we can predict from this critical quantities of rate of Helmholtz free energy, rate of entropy production or dissipation. We highlight possible caveats owing to the nonlinear nature of the equations. In the thermal runaway example discussed above we have already illustrated the basic concept underlying linear stability for a critical value. The localization problem is understood mathematically as an unstable solution of the system of partial differential equations. In a linear stability analysis this problem is tackled by simplifying to linear ordinary differential equations.

The problem solved hence is a linearized solution to what is in principle a nonlinear matrix problem. An example in mechanics would be, where a tangential stiffness matrix is a function of the nodal displacements. We look for the unknown increment of nodal displacement leading to bifurcation. This tangential stiffness is equal to a residual nodal force. If the determinant of the stiffness matrix becomes zero a bifurcation is detected. It is often pointed out that the neglect of the nonlinear terms allows establishment of a sufficient criterion for instability but not of a necessary criterion. In addition this analysis only allows us to detect a bifurcation but does not allow the prediction of how the system behaves far from this equilibrium point. The analysis is conventionally extended to derive Lyapunov functions.

Any local equilibrium state can be analyzed by means of local linear approximation to the underlying differential equation such as Equation (7). This can be done to assess stability of any equilibrium state in an extended linear thermodynamic framework. This is indeed the great achievement of Prigogine and the group from Brussels (Glansdorff *et al.*, 1973). The theory was, however, only designed for investigation of thermodynamic stability of arbitrary mass-action kinetic networks in which the reaction velocity is assumed to be proportional to the concentrations of the involved reactants. Temperature variations were not considered and the framework was reduced to a thermomechanic approximation (see Appendix A). In addition in view of the difficulty of the problem Prigogine restricted his analysis to a linearized system where Lyapunov's method can be used to derive stability far-from-equilibrium. It is important to note that on the basis of this assumption it is a linear non-equilibrium thermodynamic framework. The chosen problem of a mass diffusion is indeed (like the simple thermal diffusion problem discussed above) in its basic nature a linear thermodynamic problem. Prigogine showed the principle of minimum entropy production for the stability of non-equilibrium linear thermodynamics. We seek stability for the competing rate process of non-equilibrium, nonlinear thermodynamics. For this no analytical technique is known and we have to make use of a numerical approach.

We are left with an extrapolation from linear thermodynamics (Lavenda, 1978). If we expand terms of the nonlinear partial differential equation in a Taylor series, we may postulate that the first term of the expansion is applicable to the more general case, with the caveat that it is a sufficient but not a universal theory. We want to know how the system behaves close to the equilibrium point, e.g., whether it moves towards or away from the equilibrium point, it should therefore be good enough to keep just the linear terms. However, in a marginally stable system the higher order terms may be crucial for the phenomenon of localization and the linear theory breaks down. This means that in most cases the linearized solution will work as a good approximation to the nonlinear case, however, in some it will not work.

In order to evaluate what happens after the instability criteria are fulfilled, we cannot hence use the theory proposed by Prigogine of least rate of dissipation as a generalized basis. It only applies for the case of linear far-from-equilibrium thermodynamics. For the more general case, nonlinear partial differential equations are necessary to describe additional feedback mechanisms. Under these conditions we may not be able to describe the evolution of the system from extremum principles of entropy production. It becomes apparent from this discussion that there exists to date no universal analytically tractable theory for localization for such a case. We are left with a numerical assessment of the case and have to solve explicitly the feedback between entropy production, thermal variation and changes in Helmholtz free energy. For this system we use the Finite-Element method to derive the far-from-equilibrium behavior of the system for small perturbation away from equilibrium of a local equilibrium volume element. In the following we show how the entropy production may be calculated more explicitly.

2.4. *Calculating the Rate of Entropy Production*

The rate of entropy production can be calculated more explicitly from the above. We first expand the rate of Helmholtz free energy production using the chain rule

$$\dot{\psi}_{T,\varepsilon_{ij}^{el},\{\alpha_k\}} = \left(\frac{\partial\psi}{\partial T}\right)_{\varepsilon_{ij}^{el},\{\alpha_k\}} \dot{T} + \left(\frac{\partial\psi}{\partial\varepsilon_{ij}^{el}}\right)_{T,\{\alpha_k\}} \dot{\varepsilon}_{ij}^{el} + \left(\frac{\partial\psi}{\partial\alpha_k}\right)_{T,\varepsilon_{ij}} \dot{\alpha}_k, \tag{8}$$

where the subscripts of the partial differential equation are constant and we use Maxwell's relations (Poulet and Regenauer-Lieb, 2009) which follow from the second law of thermodynamics

$$s = -\left(\frac{\partial\psi}{\partial T}\right)_{\varepsilon_{ij}^{el},\{\alpha_k\}}, \tag{9}$$

$$\sigma_{ij} = \rho\left(\frac{\partial\psi}{\partial\varepsilon_{ij}}\right)_{T,\{\alpha_k\}}, \tag{10}$$

and define the specific heat from

$$c_\alpha \equiv -T\left(\frac{\partial^2\psi}{\partial T^2}\right)_{\varepsilon,\{\alpha_k\}}, \tag{11}$$

also from Equation (9)

$$\dot{s} = -\frac{\partial^2\psi}{\partial T^2}\dot{T} - \frac{\partial^2\psi}{\partial T\partial\alpha_k}\dot{\alpha}_k - \frac{\partial^2\psi}{\partial T\partial\varepsilon_{ij}^{el}}\dot{\varepsilon}_{ij}^{el}, \tag{12}$$

$$T\dot{s} = c_\alpha\dot{T} - T\frac{\partial^2\psi}{\partial T\partial\alpha_k}\dot{\alpha}_k - T\frac{\partial^2\psi}{\partial T\partial\varepsilon_{ij}^{el}}\dot{\varepsilon}_{ij}^{el}, \tag{13}$$

substituting Equations (5) and (7) into Equation (13) we obtain the classical energy equation with additional feedback terms that arise due to the thermal–mechanical couplings.

$$\rho c_\alpha \dot{T} = \sigma_{ij}\dot{\varepsilon}_{ij} - \rho \frac{\partial \psi}{\partial \varepsilon^{el}_{ij}} \dot{\varepsilon}^{el}_{ij} - \rho \frac{\partial \psi}{\partial \alpha_j} \dot{\alpha}_k + \rho T \frac{\partial^2 \psi}{\partial T \partial \varepsilon^{el}_{ij}} \dot{\varepsilon}^{el}_{ij} + \rho T \frac{\partial^2 \psi}{\partial T \partial \alpha_j} \dot{\alpha}_k + r_i - div\, q_i. \quad (14)$$

We are here describing the additional feedback terms in the order of their appearance. The first term on the right-hand side $\sigma_{ij}\dot{\varepsilon}_{ij}$ minus the second and third term on the right is the deformational power that is converted into heat

$$\sigma_{ij}\dot{\varepsilon}_{ij} - \rho \frac{\partial \psi}{\partial \varepsilon^{el}_{ij}} \dot{\varepsilon}^{el}_{ij} - \rho \frac{\partial \psi}{\partial \alpha_j} \dot{\alpha}_k = \chi_{(t)} \sigma_{ij} \dot{\varepsilon}^{diss}_{ij}. \quad (15)$$

The second term describes the elastic power not converted into heat, and the third term other microstrain processes that are not converted into heat such as those caused by chemical strain or other microstructural modification processes or recoverable processes. All nonthermal physical deformation processes are hidden in the second and third terms. We expect that an explicit treatment will become very important for future studies. The shear heating term is well known in the literature but normally expressed in a simpler fashion shown in Equation (15) through the introduction of the nondimensional factor χ, which is the Taylor-Quinney heat conversion efficiency with permissible values between 0 and 1. In addition only the dissipative strain rates $\dot{\varepsilon}^{diss}_{ij}$ such as those from creep or plastic deformation are considered in classical nonthermodynamic formulations. Since most experimental creep laws are reported for steady state and follow from the postulate of maximum entropy production for quasi-steady state (Appendix A), the heat conversion at steady state is expected to be very efficient. Values for most materials are indeed between 80–95% conversion efficiency (Chrysochoos and Belmahjoub, 1992). The Taylor-Quinney coefficient is often ascribed to unity and neglected.

The fourth term on the right-hand side of Equation (14) describes thermal elastic coupling. In the nonthermodynamic literature this is often neglected or replaced by the scalar or tensor-valued volumetric or scalar linear thermal-elastic expansion coefficient λ and the third term simplifies to

$$\rho T \frac{\partial^2 \psi}{\partial T \partial \varepsilon_{ij}} \dot{\varepsilon}_{ij} \equiv \lambda T_{equ} \dot{p}, \quad (16)$$

where p is the pressure and T_{equ} is a thermodynamic equilibrium temperature change. The fifth term on the right-hand side of Equation (14) is the thermal-mechanical coupling term through any additional state variable. Assuming for an example a two-phase thermodynamic system with the fractional volume ξ of one phase the coupling term is equivalent to the latent heat release upon phase change is

$$L = \rho T \frac{\partial^2 \psi}{\partial \xi_i \partial T} \dot{\xi}_i. \tag{17}$$

Equation (14) collapses for this simple case

$$\rho c_p \dot{T} = \chi_{(t)} \sigma_{ij} \dot{\varepsilon}_{ij}^{diss} + \lambda_{th} T_{equ} \dot{p} + L + r_i + \rho c_p \kappa \nabla^2 T \tag{18}$$

where κ is the thermal diffusivity. This equation is also known as the energy equation and it is typically solved for each representative volume element by finite-element analysis. In the numerical solution technique the question whether the volume element for discretization is in thermodynamic equilibrium for the chosen time step needs to be evaluated carefully (Regenauer-Lieb and Yuen, 2004).

The above-described form of the energy equation is indeed what is used in some modern modeling approaches which we will discuss in the following. If solutions are obtained through full coupling of the energy equation with the momentum and continuity equation (Appendix B), these calculations are consistent with the thermodynamic theory presented here. However, the thermodynamic theory provides additional useful tools for the assessment of instabilities such as extremum principles in entropy production to be described in a forthcoming contribution. Its biggest advantage, however, is the provision of a basic framework for cross-scale modeling, thus allowing a link of geodynamics with seismological simulations. Another advantage, relevant for seismology, is that material properties can be derived as energetically self-consistent time-dependent parameters.

3. *Instabilities and their Natural Expression*

Earthquakes are instabilities that are always manifested in an unexpected manner because of the unpredictable nature of the nonlinear material properties in the shallow portion of the crust, where a critical temperature separates a stable creeping regime from a domain where fractures can readily develop. This temperature is known in the literature (e.g., Morris, 2008), as the brittle-ductile transition temperature and holds the key to our understanding of the critical interplay between fracture mechanics and earthquakes. The depth extent of rupture in large continental earthquakes is found to be limited in the temperature regime approximately by the 570 K isotherm (Strehlau, 1986). This is well within the estimated (Regenauer-Lieb and Yuen, 2006; 2008) range of onset of creep between 450 and 500 degrees K for quartz. The far-reaching concept that the earthquake mechanism is possibly rooted in the ductile realm was first introduced in the classical work of Hobbs *et al.* (1986) and the recent work of John *et al.* (2009).

Natural expressions of these instabilities can be found in geological field evidence, which includes paleo-earthquakes, ice-quakes, landslides, pseudotachylytes found in fault zones and the phenomenon of grain-size reduction, and laboratory evidence for different types of faulting. In this work we will also discuss the potential danger of ice-quakes caused by these brittle-ductile instabilities, which might exert dramatic influence on raising the sea levels over short time scales. We describe some ongoing laboratory experiments which will call our attention to the potentially important role played by volatiles, strain-localization and also by crustal phase transitions.

4. Landslides

Landslides represent an ideal example by which a suite of different dissipative processes is triggered and instability ensues. We present here an example from Vaiont, located in the Dolomite region of the Italian Alps. On October 9, 1963 a catastrophic landslide occurred. It slipped for 45 seconds with a speed of up to 30 m/s smashing a water reservoir which at the time contained 115 million m^3 of water. A wave of water was pushed up the opposite bank and destroyed the village of Casso, located 260 m above the lake level. Other villages further downstream were also erased and 2500 lives were lost.

The slip rates of the landslides were carefully monitored before and during the event and the report MÜLLER (1964) concludes that "…the interior kinematic nature of the mobile mass, after having reached a certain limit velocity at the start of the rock slide, must have been a kind of thixotropy. This would explain why the mass appears to have slid down with an unprecedented velocity which exceeded all expectations. Only a spontaneous decrease in the interior resistance to movement would allow one to explain the fact that practically the entire potential energy of the slide mass was transformed without internal absorption of energy into kinetic energy. Such a behavior of the sliding mass was beyond any possible expectation; nobody predicted it and the author believes that such a behavior was in no way predictable…"

VEVEAKIS *et al*. (2007) were able to unravel the nonlinear physics underlying the unprecedented speed of the event. The authors test the working hypothesis that slip was localized in a clay-rich water-saturated layer. They propose shear heating as the primary mechanism for triggering the long-term phase of accelerating creep, and model the creeping phase using a rigid block moving over a thin zone of high shear strain rates including shear heating. Introducing a thermal softening and velocity strengthening law for the basal material, the authors reformulate the governing equations of a water-saturated porous material, obtaining an estimate for the collapse time of the slide. They were able to calibrate the model with real velocity measurements from the slide. In order to keep the mathematical formulation tractable and to explore the limitations of the shear heating mechanism, the model was kept as simple as possible. In their impressive fit of the predicted velocity to the observed velocity (Fig. 2) they can reproduce the observations from month 5 before the catastrophic slide using the simple shear-heating hypothesis alone (VEVEAKIS *et al.*, 2007). However, the very final phase cannot explain the explosive phase of accelerated creep observed in the Vaiont Landslide. This phase

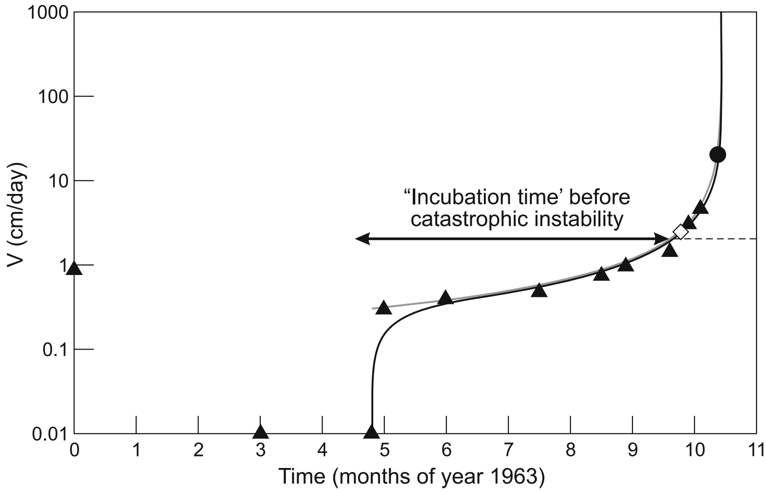

Figure 2
Slip-velocity of the Vaiont 1963 landslide (Muller, 1964) with a hyperbolic fit (Helmstetter *et al.*, 2004) shown by a grey line and the model prediction of Veveakis *et al.* (2007) depicted by a black solid line; modified after Veveakis *et al.* (2007).

occurred during the total loss of strength in the slipping zone in the last minutes prior to the slide.

Veveakis *et al.* (2007) tested the hypothesis that this second phenomenon is explained by the onset of thermal pressurization, triggered by dewatering reactions of the clay due to the temperature rise within the clay-rich layer. Their calibrated data predicted a critical temperature of dewatering of 36°C, which fits experimental data. Given that the ambient average temperature at the base of the slide is expected to be 8°C, this estimate provides an independent verification of the shear-heating hypothesis. As the mechanical dissipation through slow creep in the clay rises slowly it finally approaches a critical temperature, where dehydration reactions start. The associated water release, which increases the pore fluid pressure, subsequently leads to a catastrophic instability. For this to happen the temperature must rise up to the critical pressurization temperature, leading to an explosive pore pressure rise, up to the point of 'full pressurization': The point where pore pressure reaches the value of the total normal stress, leading to fluidization of the shear band.

We conclude from this analysis that the shear-heating hypothesis is a very robust theory to explain landslides. The physics of thermally self-accelerating localized shear in a clayey gouge is indeed a fundamental mechanism. The authors also offer solutions to the fact that not all landslides reach the catastrophic proportions encountered in Vaiont. The sudden, last minute acceleration of the slide is explained with an almost instantaneous rise of the pore pressure because of shear-heating. This critical temperature for dehydration is sufficiently high (36°C) that it may not be reached by all landslides.

A unique feature of the instability is the prolonged time before the actual onset of the catastrophic instability. When consulting Figure 2 one sees that at least 5 months before the fast instability the system appears to have been "simmering", slowly increasing its slip rate, consequently its basal temperature in the basal clay layer. The basic process underlying the instability is inherently an intrinsic thermal process. Once started, it is likely that no amount of engineering intervention (short of freezing the clay) would have been able to stop the landslide.

4.1. Ice Quakes

The concept of a 'incubation time' before onset of catastrophic instability is common in thermal feedback systems, such as combustion, where there are multiple time scales present in the phenomenon. It has been described for thermal runaway instabilities for rocks on lithospheric scale processes (Regenauer-Lieb and Yuen, 2000). The hyperbolic curves of instability are essentially the same as that of Figure 2, except that the incubation time can be hundreds thousand years before the onset of a catastrophic instability instead of five months. Regenauer-Lieb and Yuen show the same curve as Figure 2 in their Figure 9 (Regenauer-Lieb and Yuen, 2003), except that the "simmering" time-scale is hundreds of thousands of years. For ice sheets it can be between 100 years and 10 k years before the onset of ice-quakes (Yuen *et al.*, 1986). While the phenomenon of ice-quakes has been postulated by these fundamental analyses as a strict consequence of the underlying fundamental thermodynamics, seismological evidence for 184 of such events has only been provided recently (Tsai and Ekström, 2007). It is interesting to note that the size of the events appears to peak above the detection threshold, suggesting that the size of glacial earthquakes has a characteristic nature. This could indicate that ice-quakes only occur whenever a critical energy level is reached. This observation supports an energy-based instability, however, follow up numerical studies are certainly required to fully investigate this exciting phenomenon.

Schubert and Yuen (1982) also suggest massive basal melting of the Antarctic ice sheet in the form of an explosive shear-heating instability (similar to the Vaiont event). They note that the present thickness of the Antarctic ice sheet is close to the critical value for instability and perform 1-D models to investigate the effect of increased ice accumulation as finite amplitude perturbation. The difference to the case presented for Vaiont is that Schubert and Yuen do not model direct observational data which was not available at the time of writing (Tsai and Ekström, 2007). However, in principle they also consider a second style of cascading instability, which is triggered after reaching a critical energy level. This second instability would be a giant melting instability associated with a phase transition (melting). These arguments are however, for the time at best indicative. They may well be robust if the entropy were constrained in a hypothetical 1-D scenario, however, the complexity of higher dimension may throw a spanner in the works and need more processes.

4.2. Instabilities in Crustal Deformation

The Wenchuan earthquake (May 12, 2008) in Sichuan, China (Parsons *et al*., 2008) has reminded us once again that earthquakes happen unexpectedly and are a very nonlinear process. The earthquake exhibits its inherent stochastic nature with the aftershocks forming a migratory pattern toward the northeast. The unusual rupture mechanism of this earthquake has a complex strike-slip and a strong thrust component in a continental setting. Many of the nonlinear physics associated with earthquake genesis are not yet known. To date, earthquakes have been studied mainly as an elastic-deformation process, driven by kinematic boundary and initial conditions, sometimes with interacting faults (Robinson and Benites, 1995). However, realistic earthquake models require strong phenomenological and thermodynamical foundations. Such a modeling approach should consider thermodynamics within the governing equations, since the non-equilibrium nature of the entire process would influence the entire development of earthquake instabilities.

In the examples listed above we have proposed that multiple instabilities are necessary at multiple scales to fill the gap in strain rates between the fast earthquake event and the slow geodynamic creep event (Fig. 1). In the ice example we have raised concerns that a plausible mechanism in 1-D may become obsolete in 2-D or 3-D because of additional geometric complexities. The Vaiont landslide example provides sound evidence for multiple thresholds (at least two) with multiple critical values with cascading scale. The physics of natural landslides may very well be explained by exceeding the lowest critical value, which is the stability limit within which shear-heating becomes more intense than the diffusion of heat away from the shear zone. However, there is a second threshold more elevated in the energy scale, namely that of reaching a critical temperature for dehydration of clays inside the shear zone. When this critical value is reached the shear-heating instability cascades up in scale to the catastrophic event where kinetic energy must be considered. One could argue that in the deeper crust there are several such critical temperatures associated with individual phase transitions, ultimately leading to the melting instability (Kanamori *et al.*, 1998). We examine in the following section previous modeling and observational evidence for cross-scale instabilities.

In the search for the place where we have the richest instability mechanism available we are immediately attracted to the ‘semi-brittle’, brittle-ductile transition zone which from laboratory extrapolations is postulated to cover a significant fraction of the crust (Kohlstedt *et al.*, 1995). The brittle-ductile transition is an area where we can have a dual material behavior. The rock can fracture in a violent brittle manner but at the same time it can creep in a ductile manner. By its very nature, deformation within the brittle-ductile transition must hence be able to cross the scales from geodynamics to seismology and observations reported in Figure 1 might be rooted here (Strehlau, 1986). In a thermodynamic sense the brittle-ductile transition also is prominent because it is the area in the crust where the maximum dissipation occurs. Based on the sum of the arguments laid out above we conclude that the brittle-ductile transition holds the key to the mechanisms promoting earthquakes.

Unfortunately, we know very little about the mechanical behavior of the brittle-ductile transition. To date there are only very few modeling approaches oriented in this direction. Pioneering work for investigating the relation between damage by brittle cracking and aseismic creep has been proposed (LYAKHOVSKY *et al.*, 2005; LYAKHOVSKY and BEN-ZION, 2008), the role of crustal phase transition and grain size reduction have been investigated by (GUEYDAN *et al.*, 2001; 2004) and the feedback between brittle instabilities by thermal–expansion and shear-heating and ductile instabilities by shear-heating and accelerated creep have been presented (REGENAUER-LIEB and YUEN, 2006, 2008). The latter point, thermal–mechanical coupling via shear-heating in fault zones has been stressed many times since DAVE GRIGGS (1969) as a key to the understanding of geodynamical processes. However, there is, to date, no clarity on the role of the different feedback mechanisms in the crust. We argue that an understanding of the zone where transitions between brittle and ductile failures occur is critical to understanding both earthquakes on the short time scale and plate tectonics over longer time scales of millions of years. We also performed microscale observations aimed at reconciling the brittle and ductile deformation regimes (FUSSEIS *et al.*, in prep).

We investigated an exposed midcrustal section of the Redbank shear zone in Australia. Microstructural and geochemical analyses have shown that the rocks have been subject to shearing at greenschist-facies metamorphic conditions in the presence of an aqueous fluid phase (FUSSEIS *et al.*, 2009). Deformation was mostly accommodated by viscous-grain-boundary-sliding combined with creep fracturing. We found evidence for the formation of creep failure planes (Fig. 3), which form when a critical density and size of cavities promotes localized failure; cf. (DIMANOV *et al.*, 2003). While cavity growth is controlled by a creep mechanism (viscous grain boundary sliding), cavity interconnection and failure is a spontaneous nonstable process. The fact that the potential failure planes, which are preserved in our samples in a 'healed condition', are rather limited in extent suggests that failure in the presented case never reached catastrophic dimensions. However, our observation might be a possible key to bridge the gap in strain rates (Fig. 1) controlling the earthquake cycle at the base of the seismogenic zone, which underlie the Wenchuan earthquake.

5. Discussion

Future work in modelling of earthquakes, ice-quakes and landslides shares many of the same characteristics and we should appreciate this commonality. It requires us to put in more feedback processes to make them realistic. If we want to advance a step in joining geological, geophysical and numerical observations for reproducing shorter time scale instabilities, it cannot be as simple as before, as indeed pointed out in the work by LIU *et al.* (2007).

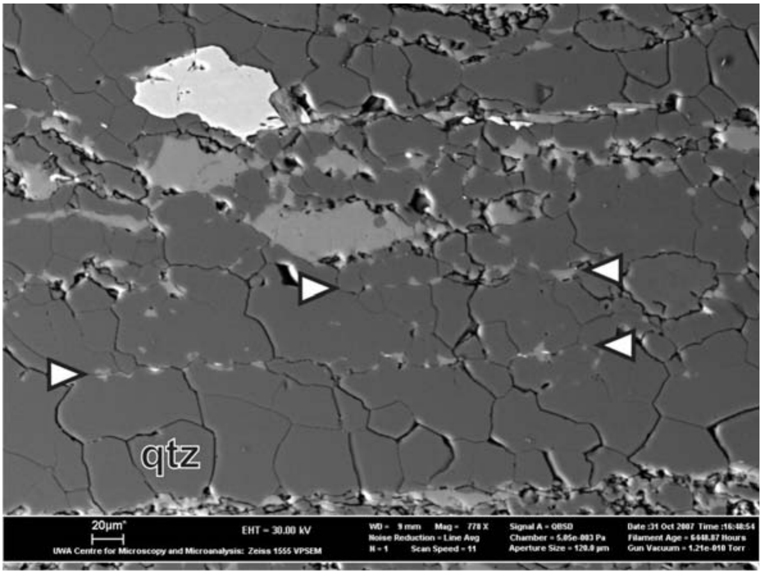

Figure 3
Back Scattered Electron image from the central portion of a natural shear zone shown in the Redbank, Australia. Microcavities (in black) decorate grain boundaries. Aligned grain boundaries mark a possible midcrustal failure plane in a quartz band (between white arrows). The failure plane is also traced by secondary K-feldspar (white) that precipitate during syn-kinematic fluid circulations. See FUSSEIS *et al*. (2009) for more details.

We propose that future numerical models always should be complemented and benchmarked by direct seismological/geological/geodynamics observations in a method of multi-scale data assimilation. This multi-scale data assimilation is crucial since, as we have pointed out, there is currently no analytical theory available that allows a prediction of the importance of the individual feedback mechanism for localization. It is hence important to build up a catalogue of important threshold values for the onset of the particular feedback and map these feedbacks to a given scale of observation. We have discussed in this paper only numerical models for thermal processes. We have, however, also put forward as a modeling challenge geological evidence for feedback processes that are not entirely thermally based. The microstructural analyses in Figure 3 point to the prominent role of the formation of microcavities in conjunction with fluid flow and dynamic permeability in the ductile crust. This observation may play a fundamental role for the strain-rate gap between geodynamics and earthquakes (Fig. 1). There may be many more processes that are unexamined yet.

Thermodynamics also allows a fair degree of simplification for these processes and provides an alternate route to assess the validity of 'brute force' modeling results. Thermodynamic approaches can be used to combine laboratory work (RYBACKI *et al.*, 2008) on natural specimens with forward numerical modelling in fault simulations and micro- to -mesoscale structural observations (FUSSEIS *et al*. in prep). In this paper we have reviewed what we believe to be solid evidence for thermodynamic instabilities that involve the consideration of the parameter temperature as a free variable. These are landslides and ice-quakes. We have not touched upon the many questions open regarding the generation of pseudotachylytes, melt formation and large-strain localization structure in dynamically recrystallizing faults such as Glarus (SCHMID, 1982) versus pseudotachylites in the Woodroffe thrust in Central Australia (LIN, 2008). We believe that at present they are providing only circumstantial evidence for their underlying process. While pseudotachylites are a very good indication of crustal melting, it is unclear whether the temperature rise is a result or a cause of localization. Melting is possible either because brittle faults are propagating into the ductile realm or they may be evidence for shear-heating instabilities propagating upwards from the ductile region. The very fact that they are happening in the brittle-ductile transition does not give preference to either mechanism.

A new vantage point that is opening a fresh view on ductile instabilities is given by inclusion of crustal phase transformations, mineral reactions and the role of chemical processes that are promoted by the role of volatiles from water and CO_2 and other crustal fluids. The thermodynamic approach proposed here provides a natural inclusion of these processes, which may help to promote shear localization in the ductile realm. To this end we are currently generalizing the approach to include chemical feedbacks (POULET and REGENAUER-LIEB, 2009). In more general terms the entropy production is the product of a thermodynamic force times a thermodynamic rate of flow. In the case that the thermodynamic flow is the flow of chemical species, the length scale for the representative volume element is the length scale of a diffusing species, which can be used to define the volume element.

Acknowledgments

We thank Yehuda Ben-Zion for encouraging us to write this article. We are grateful for stimulating discussions on shear-heating with Boris Kaus, Gabriele Morra, Sergei Medvedev, Ali Karrech and Thomas Poulet. D. A. Yuen acknowledged support from the VLAB grant given by the National Science Foundation. K. Regenauer-Lieb and Florian Fusseis acknowledge support from the Western Australian Government through the Premier's Fellowship Program. This research has been supported by the UWA research grant 12104344 and the Australian Synchrotron Research Program.

Appendix A: Thermomechanics

A much simpler framework has been proposed for Geomechanics (COLLINS and HOULSBY, 1997). The so-called "thermomechanics" approach has initially been proposed as a thermodynamic consistency check for plasticity (ZIEGLER, 1983). However, the additional advantage of the new approach is that geomechanical constitutive laws now can be derived directly from the entropy production of the underlying physical processes. This is a significant step forward, as it provides a unifying framework on the basis of physics to derive yield conditions and flow rules from the entropy production alone. The following paragraphs present a short introduction into the theory of thermomechanics. However, as will be shown in the following, thermomechanics removes key feedback potentials from Equation (7), with a strong simplifying assumption of isothermal deformation, i.e., $\dot{T}s = 0$, hence

$$\dot{u} = \dot{\psi}(T, \varepsilon^{el}, \alpha_j) + \dot{s}T, \tag{A1}$$

because there is no temperature change, there is also no heat produced in the volume and no heat flowing in and out of the reference volume and the first law becomes

$$\rho \dot{u} = \sigma_{ij}\dot{\varepsilon}_{ij}, \tag{A2}$$

since there is no adiabatic temperature change the mechanical dissipation simplifies to

$$\dot{s}T = \tilde{\Phi}, \tag{A3}$$

where the over tilde signals path dependency of the mechanical dissipation potential function. With this and Equation (A1) we have

$$\frac{1}{\rho}\sigma_{ij}\dot{\varepsilon}_{ij} = \frac{\partial\psi}{\partial\varepsilon_{ij}}\dot{\varepsilon}_{ij} + \frac{\partial\psi}{\partial\alpha^k}\dot{\alpha}^k + \frac{\partial\tilde{\Phi}}{\partial\dot{\alpha}^k}\dot{\alpha}^k, \tag{A4}$$

where α^k is the microstrain of the individual process in the representative volume element. The microstrain can be dislocation glide, dislocation climb, diffusion creep, formation of cracks, phase transitions or others. These microstrain processes can be described at the large scale of volume integration by two fundamentally different processes and we obtain

$$\sigma_{ij}\dot{\varepsilon}_{ij} = \sigma_{ij}\dot{\varepsilon}_{ij}^{el} + (1 - \chi_{(t)})\sigma_{ij}\dot{\varepsilon}_{ij}^{noheat} + \chi_{(t)}\sigma_{ij}\dot{\varepsilon}_{ij}^{diss}. \tag{A5}$$

Since the macroscopic strain is the sum of the microstrain processes α^k this formulation leads to the classical additive strain rate decomposition.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{el} + \sum \dot{\varepsilon}_{ij}^{noheat} + \sum \dot{\varepsilon}_{ij}^{diss}. \tag{A6}$$

The deformational work can be reversible and elastic, it can be plastic/viscous both through the storage of energy (e.g., in the surface energy of microcracks) or it can be released as a heat-producing or other dissipative process that does not release heat. The summation over the microstrains can be in serial or parallel. In the summation a serial description implies that the microstrain processes do not depend on another mechanism to be activated. A parallel description implies that the microstrain processes are mutually dependent. In the latter case one over the individual reciprocal strains performs the summation while in the former it is just a simple addition. These are the competing rate processes leading to localization. However, we note that the feedback terms in the energy equations have been removed through the isothermal assumption. Therefore, while this approach is instructive, it does not lend itself to earthquake modeling. We need to consider the full thermodynamic framework.

Appendix B: Coupling Temperature, Momentum and Continuity Equations

A numerical solution technique for Equation (7) requires only modest additional sophistication for modern numerical solution techniques when it is cast in its explicit form in Equation (14). In fact, the use of variational principles to find a minimum in the stored energy potential function is all that is needed. It is important to point out though that it is not enough to just solve Equation (14) separately to the underlying momentum conservation and continuity requirements. If this is done all the possible feedbacks are lost. It is necessary to derive a minimum stored energy solution of the displacements and the force balance for a given boundary condition that satisfies Equation (14). This requires full coupling of the energy equation with the momentum and continuity equations. Modern displacement finite-element methods are built around this method. In the following only a short description of an industry standard code is given, for more in-depth reading please consult the theory manual (ABAQUS/Standard, 2000).

The momentum equation describes the force and moment equilibrium. We write the equation here in integral form for the use of finite elements. Let **f** be the body force at any point within the volume and **n** the unit outward normal to a surface A. Equilibrium is achieved if the surface traction plus the body forces balance each other

$$\int_A \mathbf{n} \cdot \sigma_{ij} dA + \int_V \mathbf{f} dV = 0. \tag{B1}$$

Applying the Gauss theorem to the surface integral we obtain the equation for translational force equilibrium

$$\int_V \nabla \cdot \sigma_{ij} dV + \int_V \mathbf{f} dV = 0. \tag{B2}$$

The equation applies pointwise to an arbitrary volume and the volume integration may be dropped. The equations for rotational equilibrium can be written likewise by considering moments, however, the assumption must be made that the stress tensor is symmetric. At this stage we do not wish to abandon symmetry of stress in our numerical approach. The symmetry assumption of the stress tensor may be revisited at a later stage.

In displacement-interpolation finite-element analysis the force or moment equilibrium is routinely extended to include the continuity equation by the principle of "virtual work" here written as a principle of "virtual power". For this equation Equation (B2) is multiplied by a virtual velocity field, i.e., arbitrary vectorial test functions based on virtual displacements that satisfy the condition of continuity. These virtual displacements are used to verify whether the momentum equation is in local equilibrium.

$$\int_V \left(\nabla \cdot \sigma_{ij} + \mathbf{f}\right) \cdot \delta v \, dV = 0, \tag{B3}$$

$$\int_V \sigma_{ij} \nabla \cdot \delta v \, dV = \int_A \mathbf{n}\sigma_{ij} \cdot \delta v \, dA + \int_V \mathbf{f} \cdot \delta v \, dV = 0, \tag{B4}$$

and with the divergence of the virtual velocity field being the virtual strain rates this simplifies to

$$\int_V \sigma_{ij} \nabla \cdot \delta v \, dV = \int_V \sigma_{ij} \delta \dot{\varepsilon}_{ij} \, dV. \tag{B5}$$

The finite-element approach satisfies the momentum and continuity equation by variational principles through the minimization of the potential energy rate density functionals. According to Equation (6) minimization of the Helmholtz free energy for the smallest element size (RVE in equilibrium) is all that is needed and for any given dissipation potential the equations are closed.

References

ABAQUS/Standard (2000), 384 pp., Hibbit, Karlsson and Sorenson Inc.

Attard, P. (2006), *Theory for non-equilibrium statistical mechanics*, Phys. Chem. Chem. Phys. *8*, 3585–3611.

Chrysochoos, A. and Dupre, J. P. (1991), *Experimental analysis of thermomechanical coupling by infra-red thermography, in Anisotropy and localization of plastic deformation*, Proc. Plasticity '91, The third Internatl. Symp. on plasticity and its Current Applications (eds. Boehler;J-P and A. S. Khan, pp. 540–543, (Elsevier, London 1991)

Chrysochoos, A. and F. Belmahjoub (1992), *Thermographic analysis of thermomechanical couplings*, Archives Mechanics *44*(1), 55–68.

Clausius, R. (1865), *Über verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wärmetheorie*, Annalen der Physik und Chemie, 125, 353–400.

Collins, I. F., and G. T. Houlsby (1997), *Application of thermomechanical principles to the modelling of geotechnical materials*, Proc. Roy. Soc. London, A *453* 1964, 1975–2001.

Dieterich, J. H. (1979a), *Modeling of Rock Friction .1. Experimental Results and Constitutive Equations*, J. Geophys. Res. *84*(NB5), 2161–2168.

Dieterich, J. H. (1979b), *Modeling of Rock Friction .2. Simulation of Pre-Seismic Slip*, J. Geophys. Res. *84*(NB5), 2169–2175.

Dimanov, A. *et al.* (2003), *Creep of polycrystalline anorthite and diopside*, J. Geophys. Res. Sol. Earth, 108(B1).

Fusseis, F. *et al.* (2009), *Creep cavitation can establish a dynamic granular fluid pump in ductile shear zones*, Nature, doi:10.1038/nature08051.

Fusseis, F. *et al.* (in prep), *Earthquakes triggered by creep failure at the brittle ductile transition*, Geology, in prep.

Glansdorff, P. *et al.* (1973), *Thermodynamics Theory of Structure, Stability and Fluctuations*, Am. J. Phys. *41*(1), 147–148.

Griggs, D. and Baker, D. (1969), *The origin of deep-mantle earthquakes*. In Properties of Matter under Unusual Conditions (ed. H. M. a. S. Fernbach), pp. 23–42, Interscience New York.

GUEYDAN, F. *et al*. (2001), *Grain-size-sensitive flow and shear-stress enhancement at the brittle-ductile transition of the continental crust*, Internatl. J. Earth Sci. *90*(1), 181.

GUEYDAN, F. *et al*. (2004), *Mechanics of low-angle extensional shear zones at the brittle-ductile transition*, J. Geophy. Res. B: Solid Earth, *109*(12), 1.

HELMSTETTER, A., *et al*. (2004), *Slider block friction model for landslides: Application to Vaiont and La Clapiere landslides*, J. Geophy. Res. *109*, B002160.

HOBBS, B. E. *et al*. (1986), *Earthquakes in the ductile regime*, Pure Appl. Geophy. *124*(1/2), 310–336.

HOULSBY, G, and A. PUZRIN, *Principles of Hyperplasticity, An Approach to Plasticity Theory Based on Thermodynamic Principles*, 351 pp. (Springer, Berlin 2007).

JOHN, T. *et al*. (2009), *Generation of intermediate-depth earthquakes by self-localizing thermal runaway*, Nature Geoscience *2*, 137–140.

KANAMORI, H. *et al*. (1998), *Frictional melting during the rupture of the 1994 Bolivian earthquake*, Science *279*(5352), 839–842.

KOHLSTEDT, D. L. *et al*. (1995), *Strength of the lithosphere: Constraints imposed by laboratory measurements*, J. Geophys. Res. *100*(B9), 17587–17602.

LAVENDA, B. H. *Thermodynamics of Irreversible Processes*, 182 pp. (MacMilland Press Ltd, London 1978).

LIN, A. (2009), *Seismic slipping in the lower crust, inferred from granulite-related pseudotachylyte in the Woodroffe thrust central Australia*, Pure Appl. Geophys., in press.

LINDE, A. T. *et al*. (1996), *A Slow Earthquake Sequence on the San Andreas Fault*, Nature *383*(6595), 65–68.

LIU, M. *et al*. (2007), *Parallel computing of multi-scale continental deformation in the western United States: Preliminary results*, Phys. Earth Planet. Inter. *163*, 35–51.

LYAKHOVSKY, V. *et al*. (2005), *A viscoelastic damage rheology and rate- and state-dependent friction*, Geophys. J. Internatl. *161*(1), 179.

LYAKHOVSKY, V. and Y. BEN-ZION (2008), *Scaling relations of earthquakes and aseismic deformation in a damage rheology model*, Geophys. J. Internatl. *172*, 651–662.

MORRIS, J. (2008), *Stronger, tougher steels*, Science *320*, 1023–1024.

MULLER, L. (1964), *The rock slide in the Vaiont valley*, Felsmechanik Ingenieurgeologie, 2, 148–212.

OGAWA, M. (1987), *Shear instability in a viscoelastic material as the cause of deep focus earthquakes*, J. Geophys. Res. *92*(B1), 13801–13810.

ONSAGER, L. (1931), *Reciprocal relations in irreversible processes*, Phys. Rev. *38*(12), 2265.

PARSONS, T. *et al*. (2008), *Stress changes from the 2008 Wenchuan earthquake and increased hazard in the Sichuan basin*, Nature *254*, 509–510.

POULET, T. and REGENAUER-LIEB, K. (2009), *A unified mult-scale thermodynamical framework for coupling geomechanical and chemical simulations*, Tectonophysics, submitted.

RAMBERT, G. *et al*. (2007), *On the direct interaction between heat transfer, mass transport and chemical processes within gradient elasticity*, European J. Mechanics-A/Solids *26*(1), 68–87.

REGENAUER-LIEB, K. and YUEN, D. (2000), *Quasi-adiabatic instabilities associated with necking processes of an elasto-viscoplastic lithosphere*, Phys. Earth Planet. Inter. *118*, 89–102.

REGENAUER-LIEB, K. and YUEN, D. A. (2003), *Modeling shear zones in geological and planetary sciences: Solid- and fluid-thermal-mechanical approaches*, Earth Sci. Revi., *63*, 295–349.

REGENAUER-LIEB, K. and YUEN, D. A. (2004), *Positive feedback of interacting ductile faults from coupling of equation of state, rheology and thermal-mechanics*, Phys. Earth Planet. Inter. *142*(1-2), 113–135.

REGENAUER-LIEB, K., and YUEN, D. (2006), *Quartz Rheology and short time-scale crustal instabilities*, Pure Appl. Geophys. *163*(9), 1–18.

REGENAUER-LIEB, K., and YUEN, D. (2008), *Multiscale brittle-ductile coupling and genesis of slow earthquakes*, Pure Appl. Geophys. *165*(3-4), 523–543.

RICE, J. (1971), *Inelastic constitutive relations for solids: An internal-variable theory and its application to metal plasticity*, J. Mechan. Phys. Sol. *19*(6), 433–455.

ROBINSON, R. and BENITES, R. (1995), *Synthetic seismicity models of multiple interacting faults*, J. Geophys. Res. *100*(B9), 18229–18238.

RYBACKI, E. *et al*. (2008), *High-strain creep of feldspar rocks: Implications for cavitation and ductile failure in the lower crust*, Geophys. Res. Lett. *35*, L04304.

SCHMID, S. M. *Microfabric Studies as Indicators of deformation mechanisms and flow laws operative in Mountain Building*. In Mountain Belts (ed. K. J. Hsu), (Academic Press, London 1982).

Schubert, G. and Yuen, D. (1982), *Initiation of ice ages by creep instability and surging of the East Antarctic ice sheet*, Nature *296*, 127–130.

Siret, D. *et al.* (2008), *PreMDB, a thermodynamically consistent material database as a key to geodynamic modelling*, Geotechnica Acta, doi:10.1007/s11440-008-0065-0.

Strehlau, J. *A discussion of the depth extent of rupture in large continental earthquakes*. In *Earthquake Source Mechanics*, Proc. 5th Maurice Ewing Symp. on Earthquake Source Mechanics, May 1985 (ed., AGU, New York 1986).

Tsai, V. and Ekstrom, G. (2007), *Analysis of Glacial earthquakes*, J. Geophys. Res. *112*, F03S22.

Tse, S. T. and Rice, J. R. (1986), *Crustal Earthquake instability in relation to the depth variation of frictional slip properties*, J. Geophys. Res.-Sol. Earth and Planets *91*(B9), 9452–9472.

Veveakis, E. *et al.* (2007), *Thermo-poro-mechanics of creeping landslides: the 1963 Vaiont slide, Northern Italy*, J. Geophys. Res. *112*, F03026.

Yuen, D. A. *et al.* (1986), *Explosive growth of shear-heating instabilities in the down- slope creep of ice sheets*, J. Glaciol. *32*(112), 314–320.

Ziegler, H. *An Introduction to Thermomechanics* (North Holland, Amsterdam 1983).

(Received October 1, 2008, accepted March 26, 2009)
Published Online First: June 27, 2009